Raising Awareness the History of Mathematics in High School Curriculum

Leslie Bolinger-Horton, Ed.D
Professor of Mathematics
Quinsigamond Community College
670 West Boylston Street
Worcester, MA, 01606, USA.

Regina M. Panasuk, Ph.D
Professor of Mathematics Education
Graduate School of Education
University of Massachusetts Lowell
61 Wilder Street, O'Leary 526
Lowell, MA, 01854, USA.

Abstract

This article describes a portion of a large scale study which investigated the issues related to the integration of the history of mathematics into high school mathematics instruction. We argue that while the community of mathematics educators puts forth efforts to implement curriculum reform in school mathematics, an explicit discussion of the importance of the inclusion of the history of mathematics is missing from the conversation. There exists a gap between what is espoused in the professional and scholarly arena regarding possible benefits of students learning the history of mathematics and teachers' perceptions of the use of the history of mathematics in curriculum. This paper is focused on the analysis and the comparison of two philosophical positions about the nature of mathematics and mathematics education held by teachers, fallibilist and absolutist, and how these positions affect teachers' decision to integrate the history of mathematics into their classroom.

Key Words: history of mathematics (HOM), absolutists /fallibilists views on the nature of mathematics.

1. INTRODUCTION

The purpose of our large scale research study was to gain understanding of the high school mathematics teachers' perceptions and beliefs about the integration of the History Of Mathematics (HOM) into their instruction. While we documented multiple findings, this paper addresses only the comparison of two philosophical positions, held by teachers, about the nature of mathematics and mathematics education, fallibilist and absolutist, and how these positions affect teachers' decision to integrate the HOM into their classroom. Other significant findings are addressed in further publications.

Serving as a foundation of our research were several key assumptions well known to the community of mathematicians and mathematics educators. First, HOM provides a background for gaining a rich and deep understanding of the development of mathematical concepts. Second, by learning about the evolution of at least some mathematics concepts, which are inescapably linked to the individuals who, through years of sacrifice, trials and tribulations, invented and created the mathematical concepts and the language to communicate, one looks into past ages and traces the intellectual development of humankind. Third, there may exist an implicit relationship between the learning of the HOM and students' attitude toward mathematics. While there is no empirical proof for such assumptions, it seems commonsense to believe that it is beneficial to mathematics learners, particularly those who have difficulty understanding the significance and relational value of mathematical concepts, to be aware of the intellectual struggle of those who created mathematics and some facts from mathematicians' personal lives in order to appreciate the process of invention, which was followed by either public acceptance and recognition or rejection. In addition, teachers are often encouraged to teach mathematics as a social construction, an activity that makes sense through its usefulness, and it is fair to say that most teachers strive to inspire their students' *interest* in mathematics. To *value* mathematics the students should have varied experiences related to the cultural and historical aspects of evolution of mathematics so that they can appreciate the role of mathematics in the development of our society.

In fact, any elementary mathematics curriculum can be viewed as a recapitulation of the mathematics history, i.e., the occurrence of the development of successive stages resembling the series of the stages of the development of mathematics.

And the last but not the least assumption is that teacher philosophical views on mathematics as a discipline and mathematics teaching are the most critical factors, which affect their decision about curriculum and teaching methods. According to Thom (1973), "…whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (p. 204). Cooney (1988) suggested that teachers' views, beliefs and preferences about mathematics influence their instructional practice, thus we claim that the decision of whether or not to integrate the HOM into the classroom falls into this category. In other words, if the philosophical tenets of the nature of mathematics held by a teacher impact his/her pedagogy, it is reasonable to assume that the teacher's philosophy of the nature of mathematics impacts, specifically, the teacher's position on the inclusion of HOM. There is even a debate whether the notion of "using history of mathematics" should be replaced with "integrating history of mathematics" which apparently encourages the view that the HOM is inseparable from the subject itself (Siu &Tzanakis, 2004).

The importance of the HOM in the school curriculum has been emphasized by professional councils such as the National Council of Teachers of Mathematics (NCTM), National Research Council (NRC) and National Council for Accreditation of Teacher Education (NCATE), and has been supported by research studies (e.g., Siu, 2004; Weng Kin, 2008).

The National Council of Teachers of Mathematics stated, "Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects" (NCTM, 2000, p. 4). In the words of J.W. L. Glaishier (1848 - 1928), "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." Swetz (1994) suggested that "The history of mathematics supplies human roots to the subject. It associates mathematics with people and their needs. It humanizes the subject and, in doing so, removes some of its mystique" (p. 1).

We support the position that mathematics is a "cultural phenomenon" (Wilder, 1968, p.xi), and that meaningful learning of school mathematics must be facilitated by studying the cultural significance of mathematics, the role of the evolution of mathematical concepts and scientific thought. Swetz, Fauvel, Bekken, Johansson, and Katz (1995) suggested that exposure to the HOM at the high school level can have "…a profound effect! For it is at the secondary or high school level that students first experience the power of mathematics and begin to realize the wide scope of its application and possibilities" (p. 1). Barbin et al. (2000) posited:

> The conviction that the use of history improves the learning of mathematics rests on two assumptions about the process of learning. The more a student is interested in mathematics, the more work will be done; and, the more work that is done the greater will be the resulting learning and understanding. (p. 69)

Unfortunately, the historical dimension of teaching mathematics is either totally absent and/or ignored or viewed as an "exotic luxury" as Whitrow (1932) suggested. To date, very little has been done, in classrooms, to change Whitrow's sentiment. Research into teachers' perspectives of the inclusion of HOM in the classroom is scarce. Studies (e.g., Philippou & Christou, 1998; Schram, Wilcox, Lapan & Lanier, 1988; Siu, 2004; Smestad, 2009; Stander, 1989) indicated teachers' interests in and value of mathematics increased when introduced to the HOM. At the same time, these studies emphasized that teachers found no interest in *using* the HOM within the mandated curriculum. Among the reasons for the HOM being considered an exotic luxury, is the teachers' inner belief system about the nature of mathematics and how it should be taught.

Clearly, there is a gap between what is espoused in the professional and scholarly arena regarding possible benefits of students learning the HOM, the curriculum standards, which have no trace of the HOM, and teachers' views on the integration of the HOM in curriculum. Fasanelli et al. (2000) asserted, "These decisions are ultimately political, albeit influenced by a number of factors including the experience of teachers, the expectations of parents and employers, and the social context of debates about the content and style of the curriculum" (p. 1). It would seem highly probable that a primary influence in a teacher's decision of whether or not to include the HOM in the classroom is the set of government created, learning objectives provided in the national or state, curriculum requirements.

Fasanelli et al. (2000), report that many countries have a Magistrate of Education who outlines the educational goals for the entire country. Texts, as well as curriculum frameworks, are decided upon by the Magistrate. The countries such as Austria, Brazil, China, Denmark, France, Greece, Italy, New Zealand, Norway, Russia, have a national set of frameworks standards that explicitly incorporate HOM into the learning standards.

As of the writing of this paper, 46 states of the United States of America have adopted the Common Core State Standards (www.corestandards.org). The goal in the adoption of the common standards is to insure that students across the United States are provided with a curriculum that is unified in rigor and content.

An interesting dichotomy is that, according to Fasanelli et al. (2000), "The present ICMI study is posited on the experience of many mathematics teachers across the world that the history of mathematics makes a difference: that having history of mathematics as a resource for the teacher is beneficial" (p. 1), yet nowhere in the United States Common Core State Standards is there any mention of the learning of the HOM. Is this not sending a political message to the teachers that learning the HOM plays no role in learning the subject? If on a state level and a national level there exists the implicit message that the learning the HOM is not important, why would a teacher consider the contrary? Yet, as our study shows, some teachers include HOM in their instruction. And, since the integration of the HOM is not explicitly supported by major current state and national policies, the teacher's judgment will most likely be a function of their perceptions, attitudes and beliefs.

2. BACKGROUND

Through the ages, philosophers have supported the idea that mathematics embodies the highest standards for knowledge. It is accepted that while scientific theories are subject for revision, one must never worry about and question the truth of the equation 2+2=4. However, the evolution of mathematics and the inventions such as non-Euclidean geometry and Godel's demonstration that arithmetic is incomplete have raised concerns about the foundations and the nature of mathematics, and about its "immobility and mobility" (Kelisman, 1997, p.5).

The discussion centered on the philosophy of mathematics is rich with ideas and notions, yet controversial. Ernest (2004) described the philosophy of mathematics as:

> "…the branch of philosophy whose task is to reflect on, and account for the nature of mathematics. This is a special case of the task of epistemology which is to account for human knowledge in general. The philosophy of mathematics addresses such questions as: What is the basis for mathematical knowledge? What is the nature of mathematical truth? (p. 3)

There are many professed facets to the philosophy of mathematics, the nature of mathematics, and the nature of mathematics education. We choose the Ernest's (1993, 2004) epistemological perspectives on the nature of mathematics and how it applies to mathematics education. He suggested the ideology that mathematics teacher's philosophical perspective may tend toward a fallibilist or absolutist perspective, depending upon their view of the nature of mathematics.

2.1 Absolutist philosophy of the nature of mathematics

A person maintaining an absolutist view of the nature of mathematics, holds the belief "…that mathematical truth is absolutely certain, that mathematics is the one and perhaps the only realm of certain, unquestionable and objective knowledge" (Ernest, 1993, p. 3). As Ernest (2004) suggested, "Foundationalists and absolutists want to maintain that mathematics is certain, cumulative and untouched by social interests or developments beyond the normal patterns of historical growth" (p.6). Thus, from the absolutist perspective mathematics is infallible and undisputable, and produces always accurate results. Ernest (1993) stated, "For over two thousand years, mathematics has been dominated by an absolutist paradigm…far removed from the affairs and values of humanity" (p. *xi*).

The teachers who hold an absolutist view of the nature of mathematics as an absolute truth independent of human subjectivity are likely to present mathematics as value- and culture-free, as a set of timeless isolated facts, and are likely to encourage the routine tasks and procedures to be followed. They are less likely to elicit the excitement of the process of creating mathematics (Wilder, 1968, p.4), and to encourage students to use their "*symbolic initiative*" (p.5). The students thus strive practically on the "*symbolic reflex*" (Wilder, 1968, p.5), and are encouraged to utilize memorization, and mastering techniques for operating with symbols, which is in some ways a departure from the concept of evolution of mathematics and its scientific importance.

It is well known and documented that such pedagogical choice is likely to generate in students either total indifference to the subject or in many cases fear and negative reaction.

2.2 Fallibility philosophy of the nature of mathematics

The evolution of mathematics is not a 'royal road'. Saccheri, Gauss, Lobachevski, and Bolyai's work on Euclid's 5th postulate is a prime example. Mathematics is a human creation based upon observations and reason, and as such, it is open to error. Recognizing and acknowledging the fallibility of the mathematics opens an opportunity for change and development. Ernest (1993) describes the fallibilist, humanists, and relativists view of the nature of mathematics as one in which "…mathematical truth is corrigible, and can never be regarded as being above revision and correction" (p. 3). From the fallibilist perspective, mathematics is viewed through an historical and social perspective, and thus there are "cultural limitations to its claims of certainty, universality and absoluteness" (p. 6). Mathematics is an *a priori* science, and human invention which is not waiting to be discovered. The teachers who embrace the fallibilist, humanist philosophy of mathematics (Sfard, 1998; Hersh, 1997) view mathematics concepts as historical constructs, and exercise a humanized approach to the teaching and learning. They tend to encourage the students to investigate multiple representations of mathematics concepts and different relations among them, consider mathematics open to failure and revision, and are likely to recognize the great importance of the effect of studying the evolution of mathematics to appreciate reasoning struggle of the mathematicians. Table 1 summarizes the absolutist and fallibilist perspectives.

We stress that the terms "absolutist" and "fallibilist" are conveniently used to describe an ideology and certain philosophical constructs. The degree to which people agree with, and accept terms within the ideology will vary, and it is highly dubious that an 'ideal type of absolutist' or an 'ideal type fallibilist' exists. On the absolutist-fallibilist continuum the likelihood of multiple trends, groups and subgroups is high, and would probably be of most interests to philosophers of mathematics. For the purpose of this study and its practical implications, we limited our consideration to these two groups, and formed each group based on a certain degree of tendency toward each end on the continuum 'absolutist-fallibilist'.

2.3 About the nature of mathematics

The historical dimension of mathematics and its evolution as a living organism imbue the learning of mathematics with the ideas of mathematics as viable, ever changing, and socially constructed. There are numerous examples supporting the idea of evolution of mathematics (e.g., the act of counting goes back to the primitive civilizations and their predecessors; the need to record the process of division led to the creation of new type of numbers, i.e., fractions and their symbolic representation, the advancement of the numerations system led to the development of the whole new language to communicate newly *invented* and *discovered* mathematical ideas in a more precise and elegant way; etc). According to Wilder (1968), " The anthropologist George P. Murdock listed "numerals" as one of 72 items that occurred, so far as was known, in every culture known to history or ethnography" (p. 33*n*). Kiselman (1997) suggested that the development of mathematics on a superficial level has been driven by social and individual needs, however, on a "deeper level it has been driven by curiosity and an urge to act similar to the driving forces one finds in art" (p.1). For example, Kiselman points out that "nothing in everyday life leads directly to complex numbers, and they did not appear in any physical observations, but still they turned out to be essential for the formulation of the quantum-theoretical laws " (p. 3).

Due to humans' natural tendency to "*symbolic initiative*" (Wilder, 1968, p.5), it seems logical to accept that historically mathematics was born during the process of an invention of a language to communicate the patterns observed in the real world, as well as internal structures found within the body of mathematics. From anthropological perspective, such invention was possible due to humans' natural ability as well as necessity to think abstractly, to manipulate mental objects at a conceptual level, to establish connections between the ideas and communicated them. "Mathematics is something that man himself creates, and the type of mathematics he works out is just as much a function of the cultural demands of the time as any of his other adaptive mechanisms" (Wilder, 1968, p.4).

The language of mathematics has its own terms (e.g., symbols, notations, formulas) and mathematical statements, which are operated according to certain rules that have also been developed as a logical consequence of the expansion of mathematics structures and their language. The evolution of mathematics has taught us great lessons about the development of the overlapping structures within the subject of mathematics which over the time have been evolving and expanding (Steen, 1988).

To illustrate, let us consider the following example. Among common fractions there is a set of fractions whose denominators are powers of 10, which somehow behave similarly. Thus, it seems logical and convenient to *give* a new name to those fractions (i.e., decimals), and also following our 'symbolic initiative' *create* a new notation where the denominator is not explicitly seen as in common fractions, yet can be easily recognized by simple visual inspection of the notation. This new pattern and its notation (new way of recording special case of fractions with the denominators of power of 10) apparently induce new rules to act upon decimals, thus paving the way to the development of the new structures and consequently new language of describing these structures.

We emphasize that referring to mathematics as a language is critical to accentuate that mathematics is a result of social practices of people, rather than an objective realm, which is metaphysical and superhuman. The dual nature of mathematics and its evolution are subtle but critical to understand, because they might be a source of misconceptions about the nature of mathematics as a discipline, which may lead to misconceptions about teaching and learning of mathematics.

The roots of absolutist-fallibilist teacher beliefs about the nature of mathematics can be different. Whatever the teacher's philosophical perspective of the nature of mathematics is, it is likely that the teacher's classroom pedagogy reflects his/her belief, and thus plays a critical role in students' education. Teacher philosophy of the nature of mathematics and mathematics education is naturally infused into teacher pedagogy which in turn affects students' epistemological perspectives of mathematics as well as the foundation upon which new mathematical knowledge is constructed.

3. METHOD

To investigate high school teachers' perceptions of and to tap into the tacit factors that influence teachers' decision whether to include the HOM into their classroom instruction, a comprehensive scale instrument was designed. Having weighed the advantages and the disadvantages of both on-line and postal mail, we chose to disseminate a web based survey via SurveyMonkey™, a web-based software program (www.surveymonkey.com) and have the participant responses sent to the company's server for convenient yet confidential access.

3.1 Participants

This study took place within the confines of a New England State which had 372 operating public high schools including charter schools with around 2,909 mathematics teachers (State Department of Education, 2010). We used two avenues to encourage participations. All public and 7 private high school principals (total 379) were contacted via email requesting to forward to their mathematics teachers an invitation to participate in an anonymous, online survey. The number of the private high school mathematics teachers remains unknown. Only 6 principals declined the request. We also utilized our extended personal contacts and sent a bulk of e-mail messages to high school mathematics teachers requesting their participation. It might be a case that some teachers were approached by both their principles and the researchers. The invitation letters to the teachers included a description of the study and detailed instructions on how to access the survey through the link to the web site SurveyMonkey™ and contact information should they have any questions. All were encouraged to access the survey within two weeks. However, due to slow response and school break, we sent follow up e-mails informing the teachers about the extension of access to the survey. A total of 367 teachers participated in the online survey, which is about 11% of all high school mathematics teachers in the state. This prompts one to accept the premise that the results of the study can be considered generalizable.

3.2 Instrument

The survey's six parts consisted of 110 items. Some items were designed by the researchers prompted by the writings of Ernest (1993, 2004). Other items were adopted with modification from surveys of previous studies (Tapia & Marsh, 2004; Dutton, 1962; Shulman, 1986; Alken, 1974; Charalambous, Panaoura & Philippou, 2009; Tzanakis et al., 2000), and some items were extracted from the NAEP Mathematics Teacher Background Questionnaire (2009).

A Likert scale, as seemingly appropriate to suit the purpose of the study, consisted of 5 declarative sentences with choice responses varying in degree from strongly disagree with value of 1, disagree, neutral, agree, to strongly agree with value of 5. Among other questions (e.g., education background, experience, the routes of obtaining their license, etc.), participants were asked to select the response that most closely related to their attitudes and beliefs.

3.3 Reliability analysis and scales formation

Prior to the collection of the data, in order to assure ease in delivery, quality of construction of the survey, and face validity a small pilot study with 12 contacts was conducted. The pilot study group and the participants of this study were mutually exclusive. We centered the analysis of the data around teachers' philosophical perspectives on the nature of mathematics and its relationship to teachers' decision on whether to integrate HOM into their curriculum. Of the 110 questions, 74 Likert (ordinal) questions were used to run a reliability analysis, which yielded a Cronbach's alpha value of .94. Then, a total of seven constructs were formed as the result of a factor analysis of all 74 Likert (ordinal) data items. For each construct, scales were developed by averaging the responses (1-strongly disagree to 5-strongly agree) for the variables that loaded into the corresponding construct. Two of the seven constructs, related to the absolutist /fallibilist views of the nature of mathematics' ideology described by Ernest (1993, 2004), are described below.

The construct *absolutist* was formed by the average response of the Likert scale values for the following statements:

1. Mathematics is a system of knowledge with precise results and faultless procedures.
2. Mathematics is a system of knowledge with rigid definitions and structured procedures.
3. Mathematical knowledge consists of certain and unchallengeable truth.
4. The mathematical laws/properties/rules are always true.
5. There is a conventional manner in which mathematical ideas should be recorded.
6. Mathematics follows a hierarchical sequence.

Items with insufficient Factor Loadings included: a). Mathematics is a set of unrelated algorithms, procedures, rules, and theorems; b). Mathematics is a system of concepts that represent the physical world.

The construct *fallibilist* was formed by the average response of the Likert scale values for the following statements:

1. Mathematics is perpetually open to revision.
2. Mathematics serves certain human needs.
3. The development of mathematics is associated with certain human needs and problems.
4. Mathematics is an evolving body of knowledge necessary for daily activities.
5. Mathematics is an integral part of human culture.

Items with insufficient Factor Loadings included: a). Mathematics is fallible (i.e., imperfect); b). A mathematical theory, even if proven, is always open to change; c). Mathematics is a product of human creativity.

3.4 Data analysis

Two sets of data were compared and analyzed. The first set was related to the idea of the integration of HOM, the second set was related to teachers' view of the nature of mathematics and mathematics education.

Thirty (30) participants (≈ 8% of the total $N = 367$) left blank the question of whether they include the HOM. One hundred and thirty three (133) teachers (≈ 39 % of the total number of participants who responded to the question, $N = 337$) reported that they do not include HOM into their instruction. Two hundred and four (204) teachers (≈ 61% of the total number of participants who responded to the question, $N = 337$) indicated that they do include HOM into classroom instruction. Among the 133 teachers who reported that they do not include the HOM, 2 did not respond to the questions pertaining to their philosophical orientation. Thus, we counted 131 teachers in total who reported not including the HOM but were identifiable by their philosophical orientation. Of the 204 teachers who indicated that they do include the HOM, 2 teachers did not respond to the fallibilist construct and there were 3 teachers who did not respond to the absolutist scale. Thus, we counted a total of 199 teachers who included HOM and were identifiable by their philosophical orientation.

Analyzing the data, we were interested in learning whether there is a relationship between a decision to include HOM and the tendency toward a certain ideology, fallibilist or absolutist. Table 2 summarizes the data.

Fallibilists: The independent samples t-test analysis indicates that the responses of the total 131 teachers who reported **not** including the HOM reflected a mean of 3.95 (1 – strongly disagree to 5-strongly agree) on the fallibilist scale, and teacher responses among the total 202 teachers who reported including the HOM reflected a mean of 4.19 on the fallibilist scale.

According to the t-test for equality of means, there is a significant difference in the mean values ($p < .01$). It suggests that teachers who have a fallibilist perspective on the nature of mathematics are more likely to include the HOM into their classroom lessons.

Absolutists: The independent samples t-test analysis for the absolutist scale indicates that the mean for the total of 131 teachers who do not include the HOM was 3.13 and for the total of 199 teachers who do include the HOM, the mean scale rating is 2.99. The t-test for equality of means indicates that this difference is *close* to significant ($p \approx .056$). While our data do not strongly suggest that teachers who have an absolutist perspective on the nature of mathematics are less likely to include the HOM into their classroom lessons, we raise a flag of awareness and suggest future research on this issue.

4. DISCUSSION

Prior research (e.g., Ball, 1988; Thompson, 1984) suggested that teachers bring to the classroom a set of philosophical beliefs, and interpretations that ultimately influence the pedagogy of the classroom. The results of our study support Lerman's (1990) finding of potential influence of philosophical views of mathematics on teaching of mathematics. Lerman suggested, "The results stand…as an indication of the possible connections…" (p. 59). If a teacher's philosophical view of mathematics affects his/her pedagogy, then any historical dimension of mathematics may either be in conflict, or harmony, with the teacher's perception of the nature of mathematics.

We speculate that if a teacher holds a fallibilist perspective in which mathematics is value laden due to human inventiveness and creativity, then it is probable that he/she acknowledges the role of humans in the historical evolution of an ever changing science and will thus, be likely to consider the historical dimension of mathematics in his/her pedagogy. It is likely that a teacher with a fallibilist perspective includes the HOM to facilitate the idea that mathematics is viable, ever changing, and socially constructed. If a teacher holds an absolutist view of mathematics in which he/she considers mathematics to be unquestionable (in terms of its truth), and removed from humanity, then it would likely be the case that a historical perspective on the evolution of mathematics would conflict with his/her philosophical perspective of the nature of mathematics.

Given that a teacher's philosophical perspectives of mathematics influences the presentation of a mathematical concepts, it is not unreasonable to be concerned that students of an absolutist teacher will gain a perspective of mathematics as a faultless science of disjointed concepts devoid of human connection, and consisting of rigid definitions, fixed procedures, and only correct answers. This does not bode well for the students learning mathematics and for the formation of their perspective of mathematics. Wilder (1968) has already non-empirically analyzed, theorized and presented the case that the evolution of mathematics is critical element of mathematics curriculum. We utilize his ideas to support our claims related to integrating evolution of mathematics into curriculum. We do not suggest replacing the mathematics curriculum with historical facts but rather incorporate some developmental ideas into the fabric of the inquiry about mathematics.

If we accept the notion that mathematics is a culture that depends on symbols (language) and investigations of relationships between them, we need to recognize that humankind original behavior was to develop the tools for communication, i.e., the language of mathematics that is primarily symbols. The "symbolic initiative" (Wilder, 1968, p.5) led to creation of the justifiable laborsaving devices such as formulas, rules, algorithms. Related to this, Wilder (1968) suggested, "…the mathematician often puts much efforts into devising them…" and "…the professional mathematician understands the purpose of what he is doing…" (p.5). However, the students who are presented with the devices, tricky gadgets, and techniques for manipulation of symbols do not understand why they work. These students are forced to memorize the tricks and thus according to Wilder (1968), "drop to the symbolic reflex level", which involves "no use of symbolic initiative" (p.5).

No one can make assertions, convincingly and with certainty, about explicit and quantitatively proven benefits of learning HOM. However, to date, there exists no empirical research that indicates that HOM is *not* beneficial to students' construction of mathematical content knowledge. On another note, if teachers are not excited about evolution of mathematics as a mysterious saga of human lives and puzzling events, it is highly unlikely, they will excite their students, and as Wilder (1968) suggested, " no amount of pedagogical training will make up for the defect" (p. 4).

5. CONCLUSION

A teacher's enthusiasm and affirmation in the importance of learning the HOM may, implicitly, affect students' attitude in mathematics as well as their affinity to the field of mathematics.

We believe that HOM would stand a greater chance of integration if teachers are better informed about different positions and philosophical trends in mathematics and mathematics education, as well as their own philosophical perspective on the nature of mathematics.

We suggest that when the teachers become in tune with their beliefs and are informed about the ways to reflecting on their practices from philosophical perspective, they might be in a better position to recognize and distinguish the controversies, arguments, and alternatives related to their practices. To consider alternatives, one has to be aware of the alternatives. We assert that raising teachers' awareness of philosophical orientations (e.g., absolutist/fallibilist), should be one of the major goals of mathematics teacher professional development and should include a study of the evolution of the mathematics concepts and their connections. Currently, the predominance of an absolutist paradigm within the classroom is host to a multitude of infractions which are interfering with students' construction of mathematical knowledge. Students are learning mathematics as isolated facts unrelated within the mathematics curriculum and among various other disciplines. Students are under the impression that mathematics has always existed or was discovered as it is usually presented in today's textbooks: clean, factual, algorithmic, and never devoid of a correct answer. Anything outside of this line of thought prompts the learners to consider they do not possess the "math gene."

In our view, there is a significant reason for students to view mathematics as a human creation that began thousands of years ago and is ever changing. Students who view mathematics as a set of discrete, unrelated topics may have difficulty in understanding the relational worth of each mathematical concept, its attachment and value to human life.

Raising awareness of the value or importance of the HOM is critical and timely necessary. Teachers must have a vested interest in the role that learning the HOM plays in students' construction of mathematical knowledge and they must become aware of the importance that their own philosophical perspective, on the nature of mathematics, has in accomplishing this goal.

REFERENCES

Alken, L. R. (1974). Two scales of attitude toward mathematics. *Journal for Research in Mathematics Education, 5*(2), 67-71.

Ball, D. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8*(1), 40-48.

Barbin, E., Bagni, G. T., Grugnetti, L., Kronfellner, M., Lakoma, E., & Menghini, M. (2000). Integrating history: Research perspectives. In J. Fauvel & J. Van Maanen. (Eds.), *History in mathematics education: The ICMI study* (Vol. 6, pp. 63-90). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Charalambous, C. Y., Panaoura, A., & Philippou, G. (2009). Using the history of mathematics to induce changes in preservice teachers; beliefs and attitudes: insights from evaluating a teacher education program. *Educational Studies in Mathematics, 71*(2), 161-180.

Cooney, T.J; Shealy, B.E. & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*(3), 306-333.

Dutton, W. H. (1962). Attitude change of prospective elementary school teachers toward arithmetic. *The Arithmetic Teacher, 9*, 418-424.

Ernest, P. (1993). *The philosophy of mathematics education* (2nd ed.). New York: The Falmer Press. (Original work published 1991)

Ernest, P. (2004). What is Philosophy of Math Education? *Philosophy of Mathematics Education Journal*, 18, October.

Hersh, R. (1997). *What is mathematics, really?* New York: Oxford University Press.

Kiselman, C. (1995) Matematiken i kulturen och kulturen i matematiken [The cultural significance of mathematics]. (M. Carlehed, Trans.) *Annales Academiae Regiae Scientiarum Upsaliensis, 31*(1997), 41-50.

Kline, M. (1972). *Mathematical thought from ancient to modern times* (Vol. 1). New York, NY: Oxford University Press.

Krathwohl, D. R., Bloom, B. S., & Masia, B. B. (1973). *Taxonomy of Educational Objectives, the Classification of Educational Goals. Handbook II: Affective Domain*. New York: David McKay Co., Inc.

Lerman, S. (1990). Alternative perspectives of the nature of mathematics and their influence on the teaching of mathematics. *British Educational Research Journal, 16*(1), 53-61.

National Assessment of Educational Progress. (2009). *Mathematics Teacher Background Questionnaire*. Available August 20, 2010, from National Center for Education Statistics Web site: http://nces.ed.gov/ nationsreportcard/pdf/bgq/teacher/BQ09_teach_math_g08.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Philippou, G. N., & Christou, C. (1998). The effects of a preparatory mathematics program in changing prospective teachers' attitudes towards mathematics. *Educational Studies in Mathematics, 35*(1), 189-206.

Schram, P., Wilcox, S. K., Lapan, G., & Lanier, P. (1988). Changing preservice teachers' beliefs about mathematics education. In C. A. Mahers, G. A. Goldin, & R. B. Davis (Eds.), *Proceedings of PME-NA 11* (Vol. 1, pp. 296-302). New Braunswick, NY: Rutgers University.

Schubring, G.; Cousquer, E.; Fung, C-I.; El Idrissi, A; Gispert, H.; Heiede, T., et al. (2000). History of mathematics for trainee teachers. In J. Fauvel & J. Van Maanen (Eds.), *History in mathematics education: The ICMI study* (Vol. 6, pp. 91-142). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Sfard, A. (1998). The many faces of mathematics: Do mathematicians and researchers in mathematics education speak about the same thing. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 491–511). Norwell, MA: Kluwer Academic.

Siu, M-K. (2004). No, I do not use history of mathematics in my class. Why? In S. Kaijser (Ed.), *History and pedagogy of mathematics: Proceedings of HPM 2004* (pp. 375-376). Uppsala, Sweden: HPM.

Siu, M-K & Tzanakis, C. (2004). History of mathematics in classroom teaching - appetizer? Main course? Or dessert? *Mediterranean Journal for Research in Mathematics Education, 3*(1-2).

Smestad, B. (2009). *Teachers' conceptions of history of mathematics*. Retrieved from http://home.hio.no/ ~bjorsme/HPM2008paper.pdf

Stander, D. (1989). The use of the history of mathematics in teaching. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 241-246). Philadelphia, PA: The Falmer Press.

Steen. L. A. (1988). The Science Patterns, *Science*, 240(4852), 611-616.

Swetz, F. J. (1994). *Learning activities from the history of mathematics*. Portland, ME: J. Weston Walch.

Swetz, F., Fauvel, J., Bekken, O. Johansson., B., & Katz, V. (Eds.). (1995). *Learn from the masters*. Washington, DC: The Mathematical Association of America.

Tapia, M., & Marsh, G. E. (2004). In B. Varner (Ed.), *An instrument to measure mathematics attitudes*. Retrieved August 20, 2010, Academic Exchange Quarterly: http://www.rapidintellect.com/AEQweb/cho25344l

Thompson, A. G. (May, 1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105-127.

Tzanakis, C., Arcavi, A., Correia de Sa, C., Isoda, M., Lit, C-K., Niss, M. et al. (2000). Integrating history of mathematics in the classroom: an analytic survey. In J. Fauvel & J. Van Maanen (Eds.), *History in mathematics education: The ICMI study* (Vol. 6, pp. 201-240). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Thom, R. (1973). Modern mathematics: Does it exist? In A. G. Howson (Ed.), *Developments in mathematical education* (2nd ed., pp. 194-209). New York: Cambridge University Press.

Weng Kin, H. (2008). Using history of mathematics in the teaching and learning of mathematics in Singapore. In *Proceedings of 1st RICE* (pp. 1-38), Singapore.

Whitrow, G.J. (1932). The importance of the history of mathematics in relation to the study of mathematics technique. *The Mathematical Gazette*, 15(220), 225-227.

Wilder, R. L. (1968). *Evolution of mathematical concepts: An elementary study*. New York: John Wiley & Sons, Inc.

TABLES

Table 1. Absolutist/Fallibilist

ABSOLUTIST	FALLIBILIST
Mathematics is a system of knowledge with precise results and faultless procedures. Mathematics is a system of knowledge with rigid definitions and structured procedures. Mathematical knowledge consists of certain and unchallengeable truth. The mathematical laws/properties/rules are always true. There is a conventional manner in which mathematical ideas should be recorded. Mathematics follows a hierarchical sequence.	Mathematics is perpetually open to revision. Mathematics serves certain human needs. The development of mathematics is associated with certain human needs and problems. Mathematics is an evolving body of knowledge necessary for daily activities. Mathematics is an integral part of human culture.

Table 2: A Comparison of the Philosophical Perspective Mean vs. Integration of HOM

	Mean	
Integration of HOM	Fallibilist	Absolutist
Yes	4.19	2.99
No	3.95	3.13
Difference in mean	0.24	-0.14